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NONLINEAR DYNAMICS OF LOW REYNOLDS NUMBER ROUND JETS: PERIODIC ATTRACTORS AND TRANSITION TO CHAOS

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Direct numerical simulations have been shown [1, 2] to provide detailed information on the dynamics and coherent structures of the near field of a spatially developing axisymmetric jet. In [1] we demonstrated the shift from helical to axisymmetric structures with increasing diametral Reynolds number in the range [200; 500]. At the upper bound of this range, the *varicose* $m = 0$ mode is the most amplified (m is the azimuthal wave-number). The development of the unsteady flow is accompanied by the well known phenomena: 2D Kelvin–Helmholtz instability, roll-up and pairing, stream-wise filaments and side–jets. The onset of the asymptotic chaotic state is preceded by vortex rings reconnection and breakdown of the large structures due to strong stream–wise filaments. A similar transition process was observed in temporal simulations by Melander *et al.* [3].

In this paper, we investigate the mechanisms leading to instationarity and transition to chaos for Reynolds numbers close to the lower bound of the mentioned range. The numerical implementation was the same as that described in [1]. The 3-D Navier-Stokes (NS) solver Nekton based on the spectral element space discretization has been used to solve the incompressible NS equations in a cylindrical domain of stream–wise length equal to 15 nozzle diameters D and diameter roughly equal to 10 D. Numerical shear in a very small domain at the nozzle is responsible for the spontaneous onset of the instationarity.

For these low Reynolds numbers, the amplification of the helical mode is responsible for the jet symmetry breaking at the primary instability. Due to the axisymmetry of the base flow there exist two linearly independent, equally amplified, unstable modes, identified as the counter-rotating helical modes $m = \pm 1$. Their mutual interaction leads, close to the instationarity threshold, to a succession of 3 different regimes:

Regime I, appearing for $0 < \epsilon = (Re - Re_{cr})/Re_{cr} < 1.2\%$, is characterized by a limit cycle dynamics with a single helical mode present in the flow; *Regime II* ($1.2 < \epsilon < 3\%$) having a limit torus dynamics generated by the presence of both $m = \pm 1$ modes with unequal amplitudes; and finally, *Regime III* ($3 < \epsilon < 4.5\%$) with, again, a limit cycle dynamics, resulting from the interaction of the two $m = \pm 1$ modes with equal amplitudes.

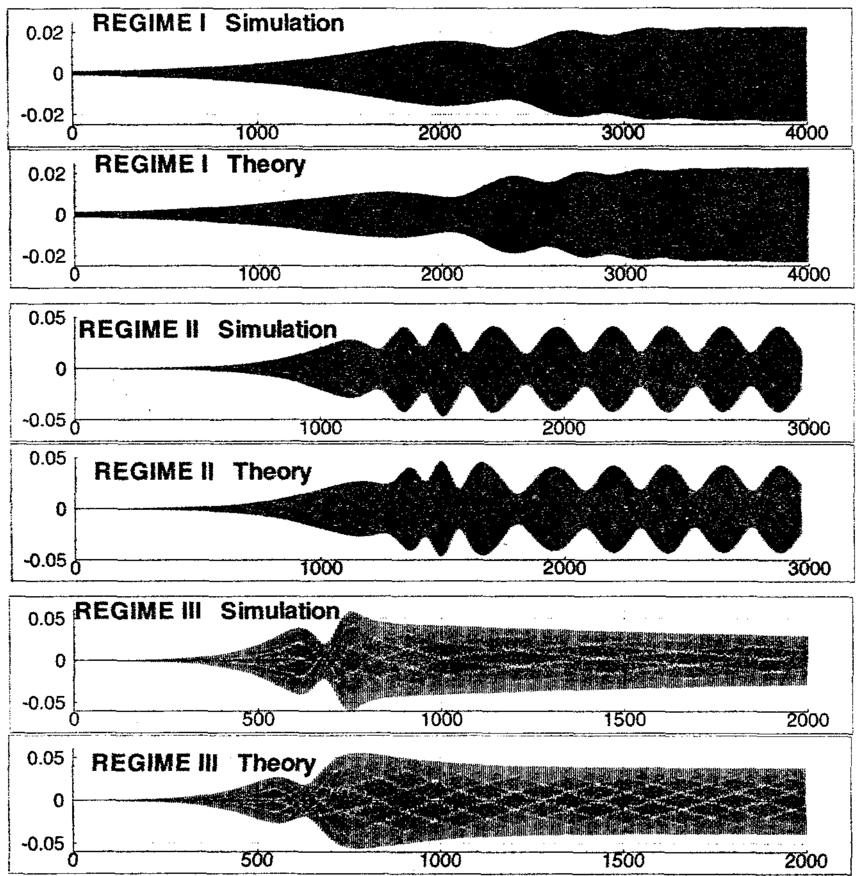

Figure 1. Azimuthal velocity at a point located in the jet mixing layer, at 2 nozzle diameters downstream. The period of oscillations is about 6 time units. Comparison between the simulated and theoretically predicted dynamics: limit cycle (regimes I and III) and limit torus (regime II).

All these stages could easily be predicted by a 5–th order weakly non-linear theory describing the interaction of the helical modes $m = \pm 1$ (see [2] for more details). The predictions of the theoretical model are in very good agreement with the results of direct numerical simulation, as illustrated in figure 1.

The chaotic state sets in about at $\epsilon = 5\%$ above the instationarity threshold. Figure 2a shows the very slow decay of the limit cycle resulting as an equilibrium of the two counter-rotating helical modes (Regime III). After very long transients, intermittent oscillations set in. The power spectra on the right side of the figure show that a new, about 50% higher, frequency appears.

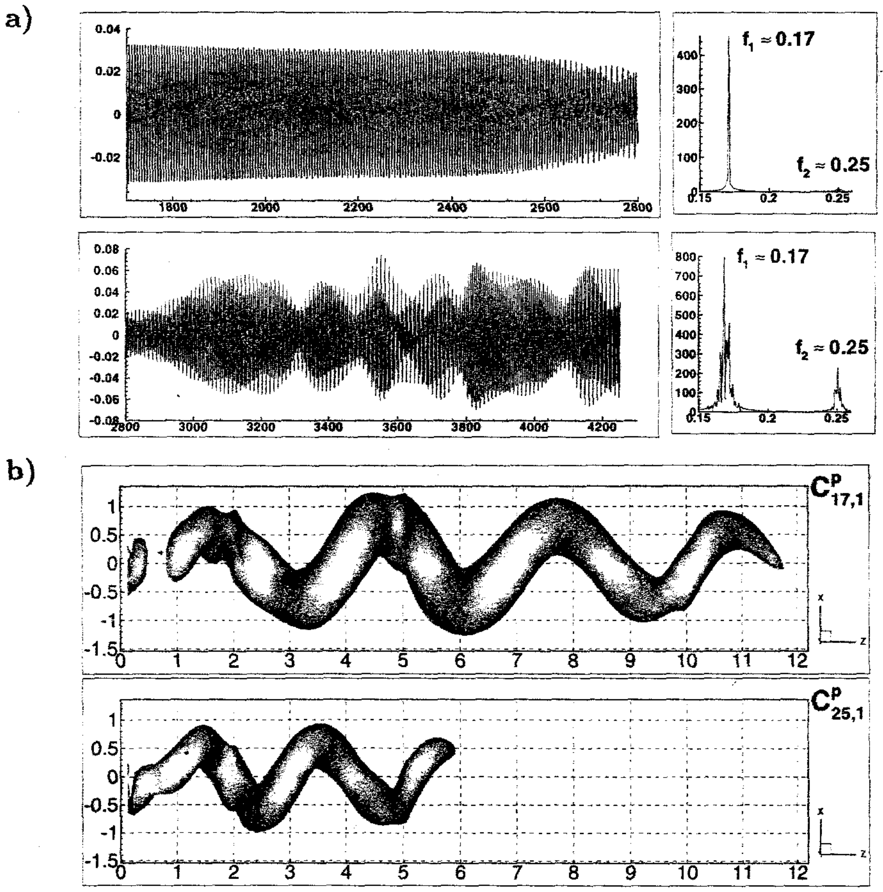

Figure 2. Direct numerical simulation at $\epsilon \approx 5\%$. a) Azimuthal velocity signal and corresponding spectra for the direct simulation (same point as in Fig. 1). b) Iso–surfaces of low pressure characterizing the amplified modes; $c_{17,1}$ (up) corresponds to the helical mode of frequency f_1 and $c_{25,1}$ (down) to the helical mode of frequency f_2.

To detect the spatial structures responsible for this new frequency, we analyzed the flow–field by the Fourier analysis proposed in [4], and applied with success in [2], to characterize the unstable symmetry breaking modes. It consists in computing temporal Fourier modes through-out the flow-field in a sufficiently large time interval. In this case, we used a time interval corresponding to 17 periods of rapid oscillations visible in Fig. 2a.

It appeared that only temporal Fourier modes c_n with $n = 1, 17$ and 25 were really significant. The mode $n = 17$, corresponding to the basic frequency (denoted f_1), being the strongest and the mode $n = 25$ (corresponding to f_2) approaching progressively the level of mode $n = 17$. The obtained temporal modes (c_n) can be further decomposed into azimuthal Fourier modes with index m. The equilibrium of the $m = \pm 1$ helical modes, characterizing the decaying *Regime III*, is thus expressed by comparing the Fourier coefficients $c_{17,\pm 1}$. The iso–pressure surfaces of dominating modes $(2Real[c_{n,m}\, e^{-im\theta}]$, with $n = 17, 25$ and $m = 1)$ are shown in Fig. 2b. It clearly appears that the second frequency is associated with another helical mode. It is interesting to note that the wavelengths of both modes have the same ratio as their periods, showing that these two modes have the same phase velocity of about 0.5, value characteristic for a jet.

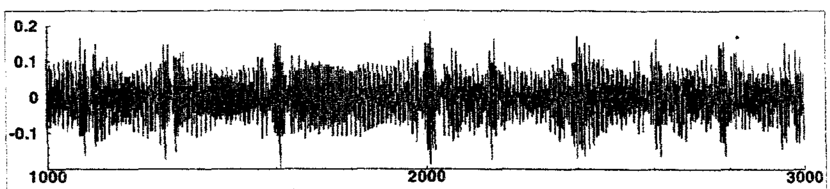

Figure 3. Direct numerical simulation at $\epsilon \approx 9\%$. Azimuthal velocity at the same point as in Fig. 1. Intermittent part of the signal.

The observed onset of chaos can be characterized as type II intermittency. A similar behavior was observed in the simulations for $5\% < \epsilon < 10\%$ above the instationarity threshold, when the chaos is completely developed (Fig. 3). The same type II of intermittency was observed at high Reynolds numbers, in the forced jet experiments of Broze & Hussain [5]. It is also interesting to note that the transition to chaos for low Reynolds numbers involves only the interaction of helical modes, while for high Reynolds numbers, the breakdown into turbulence is due to the interaction between axisymmetric and helical modes [3].

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