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MODELLING UNCERTAINTIES IN PILLAR STABILITY ANALYSIS

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ABSTRACT: Many countries are now facing problems related to their past mining activities. One of the greatest problems they have to deal with concerns the potential surface instability. In areas where a bord-and-pillar extraction method was used, deterministic methodologies are generally used to assess the risk of surface collapses. However those methodologies suffer from not being able to take into account all the uncertainties existing in any risk analysis. Through the practical example of the assessment of a single pillar stability in a very simple mining layout, this paper introduces a logical framework that can be used to incorporate the different kinds of uncertainties related to data, models as well as to specific expert's choices in the risk analysis process. Practical recommendations and efficient tools are also provided to help engineers and experts in their daily work.

KEYWORDS: Uncertainties, Risk analysis, Monte Carlo simulations, Pillar stability.

RESUME : De nombreux pays se trouvent aujourd'hui confrontés à des problèmes liés à leur histoire minière passée, et notamment à celui de l'instabilité des terrains de surface. Des méthodes déterministes sont généralement utilisées dans les zones anciennement exploitées par la méthode des chambres et piliers, pour évaluer le risque d'effondrement de surface. Cependant, ces méthodes ne permettent pas de prendre en compte les différentes incertitudes qui existent dans toute analyse de risque. Cette étude, à travers l'exemple du calcul de stabilité d'un pilier de mine extrait d'un environnement très simple, présente une démarche logique pouvant être utilisée pour incorporer les incertitudes, liées aux données aussi bien qu'aux modèles ou aux choix spécifiques des experts, dans les processus d'analyse de risque. Des recommandations pratiques et des outils efficaces pouvant permettre d'aider les ingénieurs et les experts dans leur travail de tous les jours sont également présentés.

MOTS-CLEFS : Incertitudes, Analyse de risque, Simulations de Monte Carlo, Stabilité de pilier.

1. Introduction

Many countries are facing problems related to abandoned underground mines. In France, iron ore has been industrially mined all around the Lorraine Region (North-Eastern France) from the middle of 19th to the end of the 20th century. The most common mining method was pillar extraction but a bord-and-pillar partial extraction method was sometimes used under sensitive zones to protect houses and surface infrastructures from subsidence during the exploitation. In those areas, some collapses already occurred, either during the mining operation or a long time after it. Even though numerous studies have already been carried out, researchers still go on developing and improving their works in order to understand the reasons of such collapses.

Today, one of the greatest issues of the work of state organisations having to manage the consequences of those past mining activities is to deal with the problem of potential surface instability. In the Lorraine region, INERIS and GEODERIS developed a multi-criteria analysis in

order to realise a risk map covering all the iron ore mining areas (Merad *et al.*, 2004). In The Netherlands, a deterministic method was applied successfully in the recent past to investigate the necessity of underground support measures to protect the surface from collapsing above shallow abandoned bord-and-pillar limestone workings (Bekendam, 2004). Even though those methods are perfectly efficient in their field of competence, they suffer from not being able to take into account all the uncertainties inherent to geotechnical problems.

In the past decades, literature about the use of probabilistic methods in geotechnical engineering became more and more comprehensive (Einstein, 1996; Baecher and Christian, 2003). For the purposes of any risk analysis, one may distinguish between four classes of uncertainty ranging from a very general to a very specific degree. Uncertainty can thus be attributed to (1) the scientific, economical and politic context of the study; (2) the expertise laying on deterministic human choices; (3) the use of models; and (4) the randomness and/or the lack of knowledge on data (figure 1). It can be shown from figure 1 that these four classes of uncertainty are not totally independent and that dealing with the uncertainty associated to a specific class may have impact on uncertainties of lower classes. However, the way of treating uncertainties is different between all the categories. In the higher classes, uncertainties are very difficult to deal within a quantitative way while tools exist to help considering lower classes uncertainties.

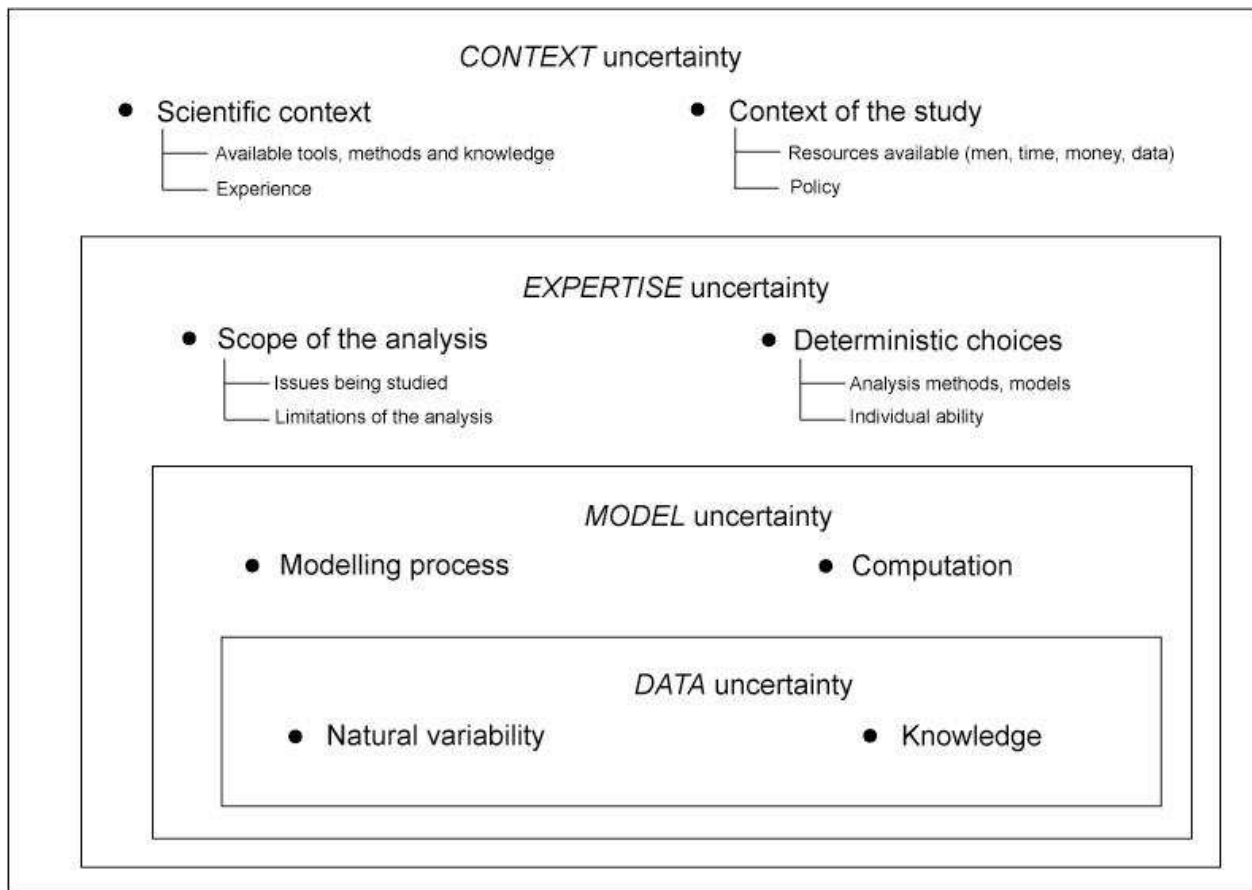


Figure 1. Possible scheme distinguishing between the different categories of uncertainty in engineering risk analysis.

In geotechnical engineering, most of the probabilistic studies that have been undertaken concern slope stability analysis. This geotechnical field indeed offers a really efficient framework for the incorporation of uncertainty into slope design (El-Ramly *et al.*, 2003) because physical phenomena are relatively well known and modelled. Nevertheless probabilistic studies are relatively rare in the mining field. Efforts have started to be made for the last couple of years in New-Zealand (Richards *et al.*, 2002) or in Great Britain (Swift and Reddish, 2002) after surface collapse events in order to

determine where further such collapses are likely to occur in the future. Even though those studies give really interesting results and make it possible to quantify the risk of future surface collapses, they do not take into account all the different kinds of uncertainties.

The study presented herein focus on the practical assessment of a single pillar stability in a very simple mining layout. It provides a methodology to incorporate different kind of uncertainties related to data, models or specific expert's choices. Different engineered recommendations will thus be given.

2. Classical deterministic approach

Bord-and-pillar method of mining is commonly used to extract ore from horizontally bedded sedimentary strata. Pillars are often left intact to provide permanent support to the undermined roof. In such a context of exploitation, economical considerations and safety assessment are the two most important issues at stake. Over the past few decades, several works about pillars have been carried out by a number of researchers and engineers either to determine optimal design pillar sizes (Salamon and Munro, 1967; Van der Merwe, 2003) or to assess long-term pillar stability (Bekendam, 2004).

The conventional methods used to assess the stability of mine pillars are based on deterministic (empiric or analytic) approaches. They usually adopt the safety factor (SF) as an indicator of the stability of the pillar. This safety factor is defined as the ratio of the pillar strength (R) over the mean vertical stress acting on pillar (S). Theoretically, a SF value greater than 1 thus means the system is stable while a SF value lower than 1 means it is unstable. In practice, a threshold value higher than 1 is generally used at the design level to incorporate uncertainties.

Several methods can be used to determine both of S and R. S is generally calculated using the Tributary Area Theory, which considers the total overburden load directly over the pillar and the portion of the galleries at its perimeter:

$$S = \gamma D \frac{(l + b_1)(L + b_L)}{lL}, \quad (1)$$

where l and L are the pillar width and length, b_1 and b_L the bord width and length, γ the average volumic weight of the overburden and D , the mining depth.

Pillar strength can be calculated by laboratory measurements on samples or by back analysis of past collapses. In the Lorraine region, several field investigations have been made after the historical surface collapses events and a back-analysis of the data lead experts to evaluate the long-term strength of the pillars to be 7.5 MPa.

In the present approach, the mining layout being studied is very simple. It consists in a grid of 3 by 3 rectangular pillars that allows to take into account the mining environment of the central pillar (figure 2). Dimensions of pillars and characteristics of the mining area have been extracted from a huge database containing information about all the Lorraine mines. The exploitation panel selected for the purpose of the study is between 130 and 150 m deep and 500 m wide. Pillars have a 12 by 8 m rectangular shape, roads are 4 m wide and the seam is 4 m high. The overburden is assumed to have a mean volumic weight of 25 kN.m⁻³. Using the tributary area theory and a constant strength value of 7.5 MPa, the pillar is assessed to be stable with a safety factor equal to 1.09.

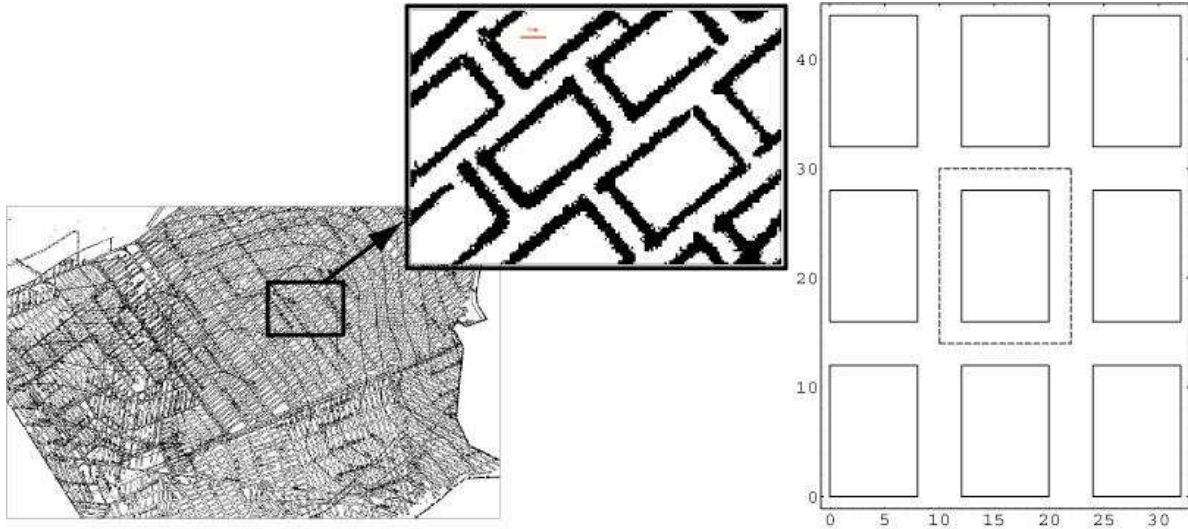


Figure 2. Extract of a mining map and selection of the nine pillar mining layout.

3. Taking into account the uncertainty on input parameters

Considering figure 1, uncertainty on input data can be linked to the spatial and temporal natural variability of mechanical and geological parameters as well as to the lack of knowledge making it difficult to assess exactly the parameter values used in models.

In our case of interest, both kinds of input data uncertainty can be found. *Physical* parameters, as γ , are affected by natural variability. They are indeed geological averaged values obtained from sample measurements. On the other side, epistemic uncertainty is predominant for parameters as the long-term strength of the pillar (R) that has been evaluated from a statistical back-analysis of several surface collapses occurred in the iron ore basin.

The determination of the nature of the uncertainty is far more difficult for the geometrical parameters such as pillar dimensions or mining depth. These are usually determined from mining paper maps archived by mining companies or regional authorities, and which are generally at a 1:5000 scale. At such a scale, a pencil line thickness on the map is equivalent to more than 1m in the mine. For classical Lorraine pillars, such a map inaccuracy may lead to a 10 % error on the values used for pillar dimensions and obviously to significant errors in the risk analysis process. It has been considered in this study that uncertainty due to the map inaccuracy belongs to *model uncertainties* as maps are models aiming to help engineers to represent the reality.

In geotechnical engineering, a very efficient way of taking into account uncertainty on input parameters is to define statistical distribution functions for each of the parameters (Kim and Gao, 1995). Unfortunately, available information is frequently poor and test data are often inefficient to characterize the statistical moments required in the distribution functions. When test data are unavailable, those parameters may be estimated from literature or judgement and experience of experts (USACE, 1999).

Once distribution functions are defined, Monte Carlo simulations may be used to propagate uncertainties in the deterministic model. The principle of such simulations is to take one random value from the distribution of each input parameter and to calculate the output value of the function. Each calculation gives one outcome. A great number of simulations allows to obtain a frequency distribution of the output values. Mean value, standard deviation or other statistical moments of the

studied parameter may thus be estimated. The greater the number of simulations, the more accurate the computed statistical moments.

For this study, a simulation method has been built allowing all the parameters to be defined by a mean value and a standard deviation. This method has been computed using Mathematica software. For the construction of the mining layout, each position of the corners of the nine pillars is randomly distributed around a mean value. In order to respect mine design reality, the assumption was made here that the roads are cut in a straight line. Map inaccuracy is supposed to lead to a 1 m error on pillar dimensions. Figure 3 presents different layouts that can be obtained from random simulations. For illustrating the difference between them, safety factors (SF) and extraction ratios (τ) were calculated for a 140 m thick overburden.

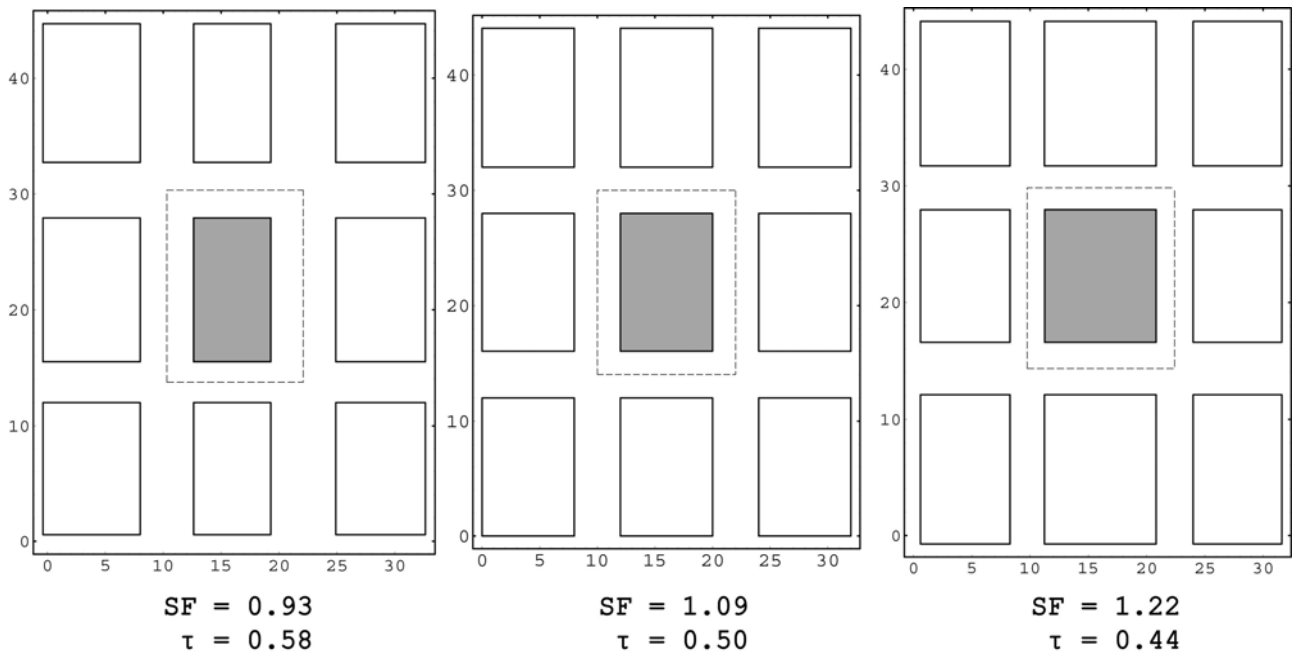


Figure 3. Three mining layouts computed from random simulations.

In order to describe statistically other input parameters, several assumptions had been made. All parameters had been characterized by a mean and a maximal error. Normal distributions were chosen to fit all the parameters. Such an assumption may be valid for parameters that have been determined from back-analysis of several past events (as R) or that are considered to be traditional mean values of sedimentary rock properties (as γ). It is more opened to criticism for parameters, as h or D, that are described in the available database by a minimal and maximal value. However, it will be shown later that the choice of a specific distribution function has only a weak influence on the result (see table 2). The maximal errors chosen for the parameters are considered to be the boundaries of the 95 % confidence interval of those normal distributions. When no information was available, maximal errors were roughly assumed to be 10 % of the mean value. Table 1 summarizes the statistical values chosen for each of the input parameters.

Table 1. Statistical values used for input parameters.

Parameter	Symbol	Mean value	Maximal error	Distribution law
Pillar length (m)	L	12	1	Uniform
Pillar width (m)	l	8	1	Uniform
Bord length = bord width (m)	b	4	1	Uniform
Pillar height (m)	h	4	0.5	Normal
Mining depth (m)	D	140	10	Normal
Average volumic weight (MN.m ⁻³)	γ	0.025	2.5e ⁻³	Normal
Long-term strength (MPa)	R	7.5	0.5	Normal
k-Strength constant (MPa)	k	4.4	(-)	Constant
Mining span (m)	A	500	(-)	Constant
Number of pillars (-)	N	31	(-)	Constant
Coefficient of geostatic stress (-)	K ₀	0.5	(-)	Constant
Young modulus of pillar material (GPa)	E _p	13	(-)	Constant
Young modulus of wall material (GPa)	E _w	5	(-)	Constant
Poisson ratio of pillar material (-)	ν_p	0.33	(-)	Constant
Poisson ratio of wall material (-)	ν_w	0.33	(-)	Constant

Using the previously described method, the statistical analysis of a large number of computed simulations made it possible to evaluate the mean value and the standard deviation of the safety factor. Figure 4 presents the obtained distribution function. From a practical viewpoint, it would be of interest to estimate p_f , the probability of ‘design failure’, defined as the probability that the computed safety factor is less than the target value of stability, i.e. 1:

$$p_f = \text{Pr ob}(SF < 1). \quad (2)$$

p_f is in this case equal to 23.5 %.

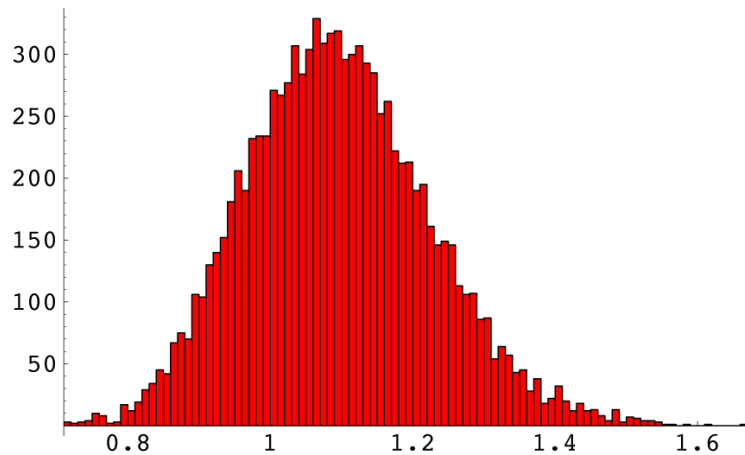


Figure 4. Distribution of the safety factor when uncertainties exist on input parameters.

While using of Monte Carlo method, an important parameter to take into account is the number of computed simulations. The greater the number of simulations, the more accurate the computed statistical moments but the longer the computation. In this study, a parametrical study has been carried out in order to investigate this parameter. Figure 5 presents for different numbers of simulations the ranges of variation of computed means of safety factors, as well as times to compute them. It can be shown that, for 10000 simulations, the computation time is still reasonable and the range of variation of the computed safety factor is less than 1 %.

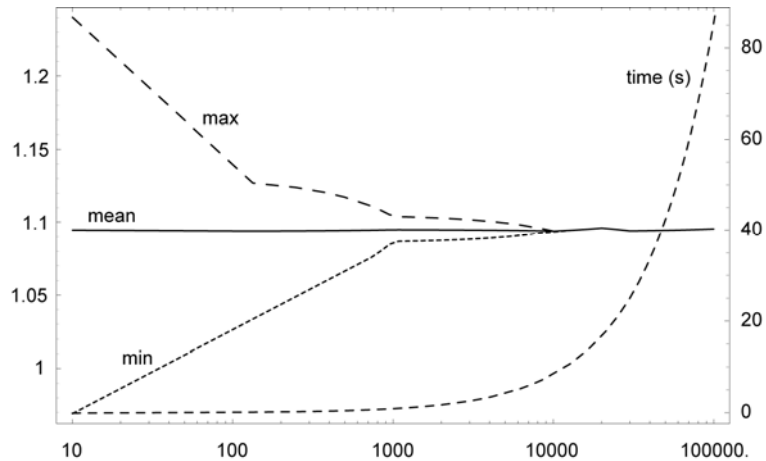


Figure 5. Range of variation of results and computation times for different numbers of simulation.

4. Taking into account the uncertainty related to the choice of a specific model

As presented earlier, uncertainty in risk analysis can also be attributed to some of the expert's choices. Assessing a risk generally requires using specific methodologies and mathematical models. For example, in our current example of pillar stability assessment, the choice has been made to compare the strength of the pillar to the stress acting on it. Two models have thus been used to estimate both of those parameters. Actually, models are no more than abstractions of the state of nature and no matter how sophisticated they are, they are unable to capture the nature entirely. The most effective approach to estimate the previously named '*model uncertainty*' would be to rely on field observations and use databases to do back-analyses. However, in geotechnical studies, it is usually very difficult to have historical observations to compare with results of model predictions and most of the times, only the uncertainty related to the choice of specific models (classified as '*expert uncertainty*') is taken into account.

In a study by Husein Malkawi *et al.* (2000), uncertainty associated with the use of different slope stability models was addressed by evaluating the relative performance of some simplified methods compared to the most rigorous and accurate one. In this study, it has been chosen to consider models as input parameters. Such a choice allows taking into account both the existence of several mathematical models and the inability of the analyst to identify the best one.

In our work previously described, two mathematical models have been used: the tributary area method has been chosen to calculate the stress acting on the central pillar of the mining layout and a constant value model has been used to evaluate the strength of a pillar. Nevertheless, even if both of the models are really convenient because simple, they also have disadvantages.

The two main drawbacks of the tributary area method are that (1) it makes the assumption that each pillar is carrying the load of a vertical rock column over it, this may only be valid for central pillars from large horizontally stratified mining environments, and (2) it does not take into account the nature of the overburden. Even though numerical simulations such as finite elements methods are today the most common methods used to compute stresses acting on pillars, they can only be used for specific sites where geology and mining geometry are known. Nevertheless, for the purpose of this study, whose aim is to be as general as possible, it has been chosen to use results from Coates' work about stresses acting on pillars (Coates, 1970). His approach makes it possible to take into account the extent of the mined area, the stress component parallel to the seam, the relative

deformation properties of pillar, roof and floor rocks, and the positions of the pillars in the mining zone. The general solution for the mean pillar stress becomes:

$$S_2 = \gamma D \left[1 + \frac{2\tau - K_0 \frac{h(1-2\nu_w)}{A(1-\nu_w)} - \frac{\nu_p}{(1-\nu_p)} K_0 \frac{h E_w}{A E_p}}{\frac{h E_w}{A E_p} + 2(1-\tau) \left(1 + \frac{1}{N} \right) + 2 \frac{L\tau(1-2\nu_w)}{A(1-\nu_w)}} \right] \quad (3)$$

$$\text{with } \tau = 1 - \frac{Ll}{(L+b)(l+b)}, \quad (4)$$

where τ is the extraction ratio, K_0 the coefficient of geostatic stress, A the extend of the mining area, N the number of pillars, E_p , E_w and ν_p , ν_w the Young modulus and Poisson ratios of pillar and wall (roof and floor) materials.

For the purpose of the study, the assumption had been made that only the parameters that are present both in equations (1) and (3) are aleatory. The impacts of those equations on the result may thus really be compared. Table 1 presents the values chosen for all the parameters. Some of these had been extracted from local databases and some are roughly estimated.

In our case of study, the use of the tributary area theory leads to a stress value of 6.87 MPa while Coates' formula gives a value of 6.66 MPa. It confirms the general belief that the tributary area theory is rather a conservative approach.

Concerning the strength of a pillar, it is now common cause that it depends on the strength of the material of which it is composed as well as its dimensions, and more specially its width-to-height ratio (Salamon and Munro, 1967; Van der Merwe, 2003). Using Van der Merwe's method on a very extensive database of failed and stable Lorraine iron ore pillars, it was found that the strength of those pillars can be expressed as follows (Cauvin, 2004):

$$R_2 = k \sqrt{\frac{w}{h}} \quad (5)$$

$$\text{with } w = 2 \frac{Ll}{L+1}. \quad (6)$$

Here, w is the equivalent width that has to be used for rectangular pillars and k a constant value that can be related to the strength of the pillar material. The determination of k lays on the assumption that, on average, pillar failure occurs for a safety factor equal to 1.

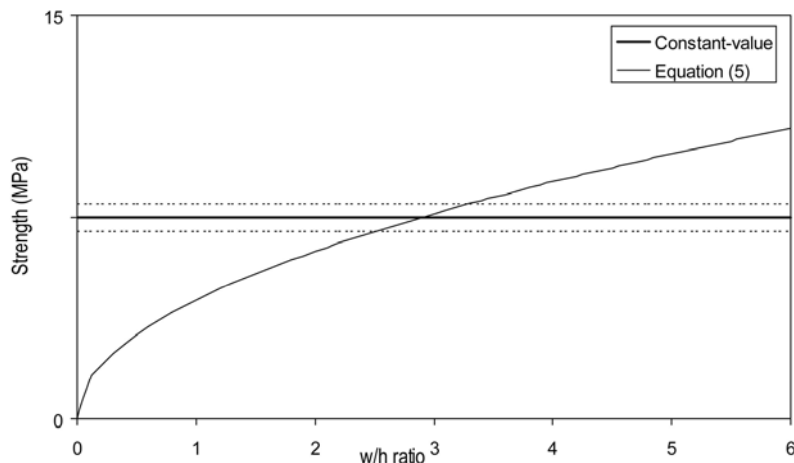


Figure 6. Comparison between the two different models estimating pillar strength.

Figure 6 shows the comparison between the two different models (constant-value versus equation (5)). For a w/h-ratio higher than 2.9, R_2 is bigger than 7.5 MPa.

Those different physical models have been implemented in the simulation method. Basically, like for input parameters, one random model is chosen among the two that exist to evaluate the strength of the pillar and the stress acting on it. The computation is then done following the same procedure as described in part 3. A degree of confidence can be chosen for models allowing taking into account decisions of experts as well as historical considerations. For example, if in a panel of 5 experts, 3 think that it is better to use a constant-strength value to express the strength of the pillar, the strength model used in the computation have a probability of 3/5 (0.6) to be this one. In one other way, in the Lorraine region the tributary area theory was originally used by miners to design the underground workings. It thus seems logical to preferably use this formula to assess the risk of pillar failure. Figure 7 shows the results obtained by the simulation method, assuming that the input parameters are perfectly known. It has been chosen here to assign a confidence degree of 75 % for the tributary area theory while the models for the pillar strength are equally “trusted”.

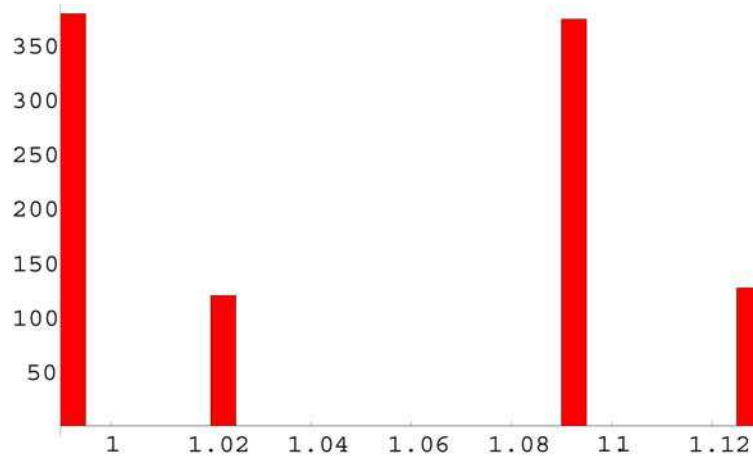


Figure 7. Computed safety factors, taking into account the existence of different models.

It can then be shown in this case that one on the four combinations gives a safety factor less than 1. This Safety Factor corresponds to the ratio between R_2 (equation 5) and S (equation 1).

Figure 8 presents the distribution function obtained for the safety factor when uncertainties on input parameters and on models are simultaneously taken into account. In comparison with the previous results, the probability of ‘design failure’ increases from 23.5 % to 34 %.

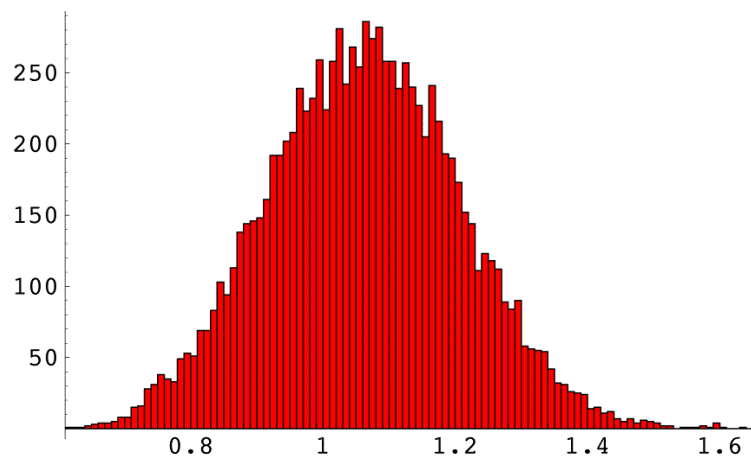


Figure 8. Distribution of the safety factor when uncertainties exist both on input parameters and models.

5. Taking into account the method used to “propagate” uncertainties

Monte Carlo simulations have been chosen as the method used to propagate uncertainty on input parameters into deterministic models. We are fortunately here in a case within which it is possible to do so. Problems with Monte Carlo simulations are that they may require huge time of computation.

However others methods exist to “transfer” the uncertainty concerning the input parameters on the output of a model. The Point Estimate Method (PEM) is one of them. It is known for its simplicity and is commonly used in geotechnical studies. One of the greatest advantages of such a method is that it is easily computable in numerical simulation processes. In PEM, each random variable is defined by two numbers: the mean value plus or minus the standard deviations of the variable. Using those points, it allows to obtain accurate approximations of the mean value and the variance of unknown output variables. For more details about the method, a detailed algorithm is presented in Isaksson (2002). The main drawback of PEM is that it does not provide any information about the shape of the distribution of the output parameter. Moreover, using complex functions or numerous correlated variables may lead to an inaccuracy of the results.

In our case of interest, Monte Carlo simulations allowed to obtain the distribution function of the pillar safety factor (figure 4). The mean value and the standard deviation of this parameter can thus be estimated to be respectively 1.095 and 0.126 after 10000 simulations. Using PEM, they have been found to be respectively 1.104 and 0.163.

6. Analysis of variance

Engineers and experts clearly ask today for efficient tools that can help them to solve the tough dilemma they face. They indeed have to give quick and accurate answers to problems full of uncertainties. One of their greatest needs is thus to reduce the uncertainty on the results they give (in this study, the standard deviation of the calculated safety factor is about 0.14) in a rapid and cheap way. For time and economic reasons, it is obvious that engineers cannot study all the sources of uncertainty but can only try to focus on some of them. The study presented herein provides a tool that makes it possible to identify the parameters whose uncertainty has the greatest impact on the result.

Table 2 presents the relative influences of five different sources of uncertainty on the total variance of the computed safety factor. The map inaccuracy and the use of human-made database, expert-made hypothesis or models (as back-analysis) to estimate parameter values may have impacts on the input parameters. But, as presented in part 4, the choice of specific models to deal with a problem may also have impacts on the result. In order to obtain Table 2, each source of uncertainty has been investigated separately. Normal and uniform distribution have been used in the Monte Carlo simulations to fit the input parameters. It can be shown that the choice of a specific distribution function has only a weak influence on the result. It can also be highlighted that in our case of interest, almost half of the total variance of the computed safety factor is due to the map inaccuracy. It has also to be specified here that the differences between the relative influences of D , γ and R on the total variance can be totally explained by the differences between the maximal error on mean ratios as the rule they play in the calculation of the safety factor is exactly the same (see equation 1).

Table 2. Relative influence of the different sources of uncertainty on the computed safety factor.

Source of uncertainty	Concerned parameters	Maximal error / Mean	Variance of the computed safety factor		Variance / Total variance (%)
			<i>Normal distribution</i>	<i>Uniform distribution</i>	
Map inaccuracy	L, l, b	~ 0.10	$7.78e^{-3}$	$9.89e^{-3}$	47
Use of database	D	0.07	$1.59e^{-3}$	$2.05e^{-3}$	9
Expert-made hypothesis	γ	0.10	$3.18e^{-3}$	$4.05e^{-3}$	19
Use of models	R	0.07	$1.37e^{-3}$	$1.77e^{-3}$	8
Choice of models	(-)	(-)	$2.82e^{-3}$		17

7. Conclusions and practical recommendations

Table 1 introduced a possible scheme to classify the different categories of uncertainty that can be encountered in a risk analysis study. This paper provides a logical framework that can be used to reduce several kinds of uncertainties. Defining statistical distribution functions for input parameters may help engineers to deal with the problem of ‘*data uncertainties*’ inherent to the study of a natural system. It can also be pointed out here that the definition of the characteristics of those distribution functions requires the expert to make some choices (evaluation of mean values, maximal errors, ...), so that the ‘*expertise uncertainty*’ is also involved. Choices had also to be made regarding the nature of the distribution functions fitting the parameters. In our study, normal and uniform distributions have both been used and it can be shown that such a choice has only a weak influence on the results. Table 2 illustrates that the variance of the computed safety factor is only slightly bigger when uniform distributions have been chosen for all the input parameters (in this case, p_f equals 24.6 %) than when normal distributions have only been used ($p_f = 21.4$ %). More generally about ‘*expertise uncertainty*’, this work may guide engineers through their analysis process in giving them a methodology to follow. It thus allows to break free from the individual ability or from the deterministic choices of the expert in charge of the analysis.

It has been said in introduction that the methods currently used to assess the risk of surface collapse do not generally integrate the taking into account of all the uncertainties inherent to the study of a natural system. In fact, it is not exactly true. Most of the times, uncertainties are introduced in the evaluation process in an indirect, if not unconscious, way. This integration is basically part of the expert’s analysis process and is affected by ‘*expertise uncertainty*’. For instance, in the example we used, finding a safety factor greater than 1 means the system has to be stable. However the value of 1.092 is very close to 1, meaning that we are thus very close to the stability. Experts usually use deterministic threshold values for safety factor (1.2/1.3/1.5/3) in order to integrate uncertainties inherent to the evaluation and to be side of the safety. Generally, the more damaging the consequences of an event, the greater the chosen threshold value.

Nevertheless, using such threshold values does not allow the expert to assess ‘*how stable the situation is*’. Taking explicitly into account uncertainties on input parameters by the use of distribution functions makes it possible for the expert to represent himself an order of magnitude of what he is talking about, e.g. in our case of interest 23.5 % of the computed safety factors are less than 1. Expressing such a quantified result now sets the problem of its interpretation. In fact, we are now talking in terms of risk and acceptance. It is now up to the stakeholders, not either to the engineers, to decide whether the situation is acceptable or not, and whether uncertainty on the result has to be reduced or not.

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